Received: *January 2, 2001*, Accepted: *January 11, 2001*

Potentials between static SU(3) sources in the fat-center-vortices model

Sedigheh Deldar

Washington University, St. Louis, MO, 63130
E-mail: sd@howdy.wustl.edu

Abstract: The potentials between static sources in various representations in $SU(3)$ are calculated based on the fat-center-vortices model of Faber, Greensite and Olejník. At intermediate distances, most distributions of the flux within vortices lead to potentials that are qualitatively in agreement with "Casimir scaling", which says that the string tension is proportional to the quadratic operator of the representation. However, at the quantitative level, violations of Casimir scaling are generally much larger than those seen in numerical simulations, indicating that additional physical input to the fat-center-vortices model is required. At large distances, screening occurs for zero-triality representations; for the representations with non-zero triality the string tension equals that of the fundamental representation. Some rather "unphysical" flux distributions can lead to violations of Casimir scaling at intermediate distances and violations of the expected ordering of representations at large distances.

Keywords: Nonperturbative Effects, Confinement, Lattice Gauge Field Theories, Phenomenological Models.

JHEP01(2001)013

Contents

1. Introduction

A variety of models have been introduced in an attempt to explain quark confinement. The center vortices model [1, 2] introduced in the late 1970's by 't Hooft is one of those attempts. A center vortex is a topological line-like (in $D = 3$ dimensions) or surface-like (in $D = 4$ dimensions) field configuration. The vortex carries magnetic flux quantized in terms of elements of the center of the group. The fluxes form narrow tubes with constant energy per unit length (surface). In order for the vortex to have finite energy per unit length, the gauge potential at large transverse distances must be a pure gauge. However, the gauge transformation which produces that potential is non-trivial. It is discontinuous by an element of the gauge center. It is the non-trivial nature of the gauge transformation which forces the vortex core to have non-zero energy and makes the vortex topologically stable. Faber, Greensite and Olejník [3] introduced fat-center-vortices to obtain confinement of both fundamental and higher representation static sources. According to the fat-center-vortices model, the vacuum is a condensate of vortices of some finite thickness. Confinement is produced by the independent fluctuations of the vortices piercing each unit area of a Wilson loop.

Faber et al. explicitly worked out the model for SU(2) and gave results for a particular flux distribution within the vortices. Here, I work in SU(3) and investigate a wide variety of vortex flux distributions. I study the existence of Casimir scaling at intermediate distances and the pattern of screening at large distances. The model confirms Casimir scaling qualitatively at intermediate distances, however, in general the model does not agree with Casimir scaling quantitatively, in contrast to the precise agreement recently found by numerical simulations [7, 8]. The lack of convexity

JHEP01(2001)013

of the potentials predicted by the model is also discussed. For completeness, I first briefly explain their model and then apply it to SU(3), using it to study the potentials between static quarks for the fundamental and several other representations.

2. The model of Faber, Greensite and Olejník

In the fundamental representation of SU(N), a center vortex linked to a Wilson loop has the effect of multiplying the Wilson loop by the gauge group center,

$$W(C) \to \exp^{2\pi i n/N} W(C)\,, \qquad n = 1, 2, \ldots, N-1\,. \tag{2.1}$$

Based on the vortex theory, the area law for a Wilson loop is due to the quantum fluctuations in the number of center vortices linking the loop. Adjoint Wilson loops are not affected by center vortices unless the vortex core overlaps the perimeter of the loops. If the vortex thickness is large enough, the fat-center-vortices model can explain confinement and the Casimir scaling of higher representation string tensions. The average Wilson loop predicted by this model has the following form:

$$\langle W(C)\rangle = \prod_{x_v} \left\{ 1 - \sum_{n=1}^{N-1} f_n \left(1 - \mathrm{Re}\,\mathcal{G}_r[\vec{\alpha}^n_C(x_v)]\right) \right\}, \tag{2.2}$$

where x_v is the location of the center of the vortex, A is the area of the loop C, and $\mathcal{G}_r$ is defined as:

$$\mathcal{G}_r[\vec{\alpha}] = \frac{1}{d_r}\,\mathrm{Tr}\exp[i\vec{\alpha}\cdot\vec{H}]\,. \tag{2.3}$$

d_r is the dimension of representation r, and $\{H_i\}$ is the subset of the generators needed to generate all elements of the center of the group. (For SU(3), λ_8 is sufficient.) The parameter f represents the probability that any given unit area is "pierced" by a vortex; i.e., a line running through the center of the vortex tube intersects the area.

The parameter $\alpha_c(x)$ is determined by the fraction of the vortex flux that is enclosed by the Wilson loop. Therefore $\alpha_c(x)$ depends on the profile of the vortex as well as the shape of the loop and the position x_v of the center of the vortex relative to the perimeter. For SU(3) $\alpha_c(x)$ is equal to $4\pi/\sqrt{3}$, if the flux is entirely inside the minimal area of the loop and it is zero if the flux is entirely outside the minimal area of the loop.

3. General features of Casimir scaling and screening

Numerical simulations [4]–[8] have shown that the potentials in SU(3) at zero temperature are linear at intermediate distances and roughly proportional to the Casimir operator. The proportionality of potential to the Casimir operator is known as

"Casimir scaling". Assuming such scaling, the potentials of each representation to that of fundamental representation would be 2.5, 2.25, 4.5, 7, 4 and 6 for representations 6, 8, 10, 15-symmetric, 15-antisymmetric and 27, respectively. The Casimir scaling regime is expected to exist for intermediate distances, perhaps extending from the onset of confinement to the onset of screening.

Screening can be understood as follows: Each representation can be labeled by the ordered pair (n, m), with n and m the original number of 3 and $\bar{3}$ which participated in constructing the representation. Triality is defined as $(n - m)$ mod 3. Screening occurs for representations with zero triality: $8 \equiv (1, 1)$, $10 \equiv (3, 0)$, and $27 \equiv (2, 2)$. For these representations, as the distance between the two adjoint sources increases, the potential energy of the flux tube rises. A pair of gluons pops of vacuum when this energy is equal or greater than the twice of glue-lump mass. (A glue-lump is the ground state hadron with a gluon field around a static adjoint source.) At large distances, the static sources combine with the adjoint(8) charges (dynamic gluons) popped out of the vacuum and produce singlets which screen. Therefore the potential between static sources is no longer R dependent. Static sources in representations 10 and 27 transform into the 8 first by combining with a dynamic gluon, and then the 8 transforms into the singlet by combining with a second gluon.

$$8 \otimes 8 = 27 \oplus \bar{10} \oplus 10 \oplus 8 \oplus 1\,, \tag{3.1}$$

$$10 \otimes 8 = 8 \oplus 10 \oplus 27 \oplus 35\,, \tag{3.2}$$

$$27 \otimes 8 = 64 \oplus 27 \oplus 27 \oplus 35 \oplus \bar{35} \oplus 10 \oplus \bar{10} \oplus 8\,. \tag{3.3}$$

Therefore, we expect the potential in representations 10 and 27 to screen only at higher energy than in representation 8. Static sources in representations with non-zero triality, $6 \equiv (2, 0)$, $15_s \equiv (4, 0)$ and $15_a \equiv (2, 1)$, transform into the lowest order representation (3 and $\bar{3}$) by binding to the gluonic $8's$ which are popped out of the vacuum:

$$6 \otimes 8 = \bar{3} \oplus 6 \oplus 15 \oplus 24\,, \tag{3.4}$$

$$15_a \otimes 8 = 42 \oplus \bar{24} \oplus 15_a \oplus 15_a \oplus \bar{6} \oplus 3 \oplus 15_s\,, \tag{3.5}$$

$$15_s \otimes 8 = 48 \oplus 42 \oplus 15_s \oplus 15_a\,. \tag{3.6}$$

15-symmetric changes to 15-antisymmetric first, so it needs to interact with the 8's (popped from the vacuum) twice to transform to 3. Screening does not occur for representations with non-zero triality, since there is no way to get a zero triality representation by crossing a non-zero one with any number of 8's. As a result, the slope of the linear potentials of the representations with non-zero triality changes to the slope of the fundamental one, and a universal string tension is observed for large R. We expect the representation 15-symmetric to require a larger value of R to approach the fundamental slope than representations 6 or 15-antisymmetric because two pairs of 8's must be popped from the vacuum in the 15-symmetric case.

JHEP01(2001)013

4. Applying the Fat-center-vortices model to SU(3)

To find the potential $V_r(R)$ in SU(3), first I need to find H_i in eq. (2.3) for each representation: 3, 6, 8, 10, 15-symmetric, 15-antisymmetric, and 27. For the fundamental representation, $H_1 = T_8 = \lambda_8/2$; where λ_8 is the diagonal Gell-Mann matrix.

I obtain T_8^r of other representations by using the tensor method. Define $\{X_r^i; i = 1, \ldots, d_r\}$, which are the basis vectors for the space on which the representation act. The corresponding generators are obtained from [9]:

$$[T_a^{D_1 \otimes D_2}]_{ix,jy} = [T_a^{D_1}]_{ij}\delta_{xy} + \delta_{ij}[T_a^{D_2}]_{xy} \,. \tag{4.1}$$

$T_a^{\prime}s$ are the group generators for representations D_1, D_2, $D_1 \otimes D_2$. The elements of T_8^r can be found by:

$$T_8^r X_r^i = \sum_{j=1}^{d_r} C_{ij} X_r^j \,. \tag{4.2}$$

To study potentials for SU(3), one needs to define an appropriate form of the function $\alpha_c(x)$ in eq. (2.2). To understand more about the effect of the vortex profile on potentials and Casimir scaling, I assume a density of flux $\rho(r)$ in an axially symmetric vortex core, where r is the radial distance from the vortex center. Let $\rho(r) = 0$ for $r > a$ so the vortex has a sharp boundary at $r = a$. Now let $\beta(x_v)$ denote the amount of the flux of the vortex contained in the region $x > 0$. Thus, $\beta(x_v) = 0$ for $x_v < -a$, and $\beta(x_v) = 4\pi/\sqrt{3}$ for $x_v > a$. For $-a \leq x_v \leq a$, $\beta(x_v)$ is determined by the integral of $\rho(r)$ over the fraction of the vortex in the region $x > 0$. Finally, let the Wilson loop have sides $x = 0$ and $x = R$. Then $\alpha_c(x_v)$, the fraction of flux within the loop, is given by:

$$\alpha_R(x_v) = \beta(x_v) - \beta(x_v - R) \,, \tag{4.3}$$

A simple choice for $\rho(r)$ that I have tried is a uniform distribution $\rho(r) = \rho_0$ for $r < a$. Another possibility with a smoother edge at $r = a$, is:

$$\rho(r) = \rho_0 \exp\left[\frac{-b}{(|r|/a - 1)^2}\right] , \tag{4.4}$$

where b is an adjustable constant and ρ_0 is fixed by the requirement that the total flux is $4\pi/\sqrt{3}$. figure 1 shows potentials obtained from this flux distribution with $b = 0.1$, $a = 20$ and $f = 0.1$. From the plot, it can be seen that, for each representation, there exists a region in which the potential is approximately linear and qualitatively in agreement with Casimir scaling. Screening occurs for representations 8, 10 and 27 while the slope of the potentials for representations 6, 15-symmetric and 15-antisymmetric changes to the slope of the fundamental representation. Note the non-convexity near $R = 8$ to $R = 20$ for all representations, and especially representations 15-symmetric and 27. Even though the fat-center-vortices model predicts some of the

JHEP01(2001)013

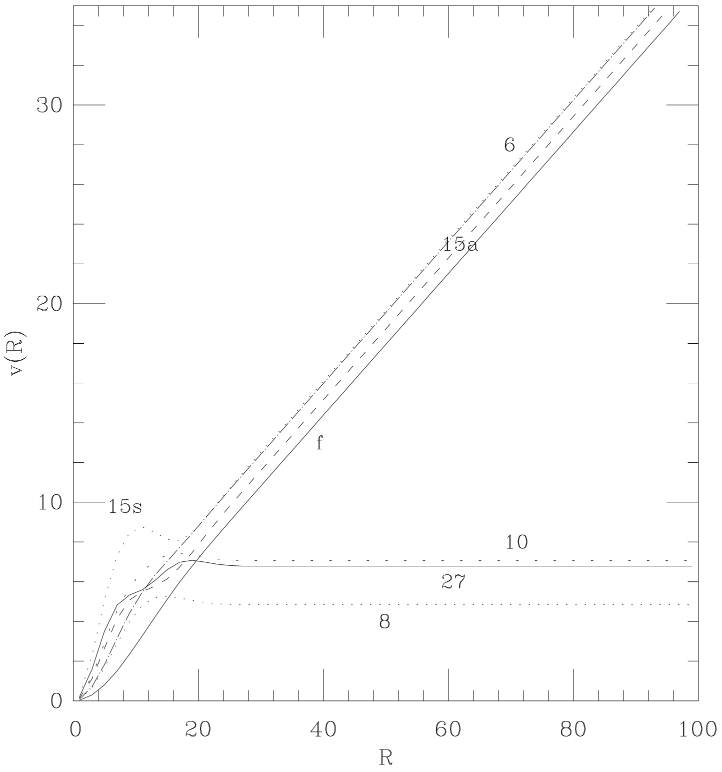

Figure 1: Potentials for different representations using density distribution proportional to $\rho_0 \exp(-.1/(|r|/a - 1.0)^2)$ for $-a < x < a$. The fundamental representation is shown by the letter "f". At intermediate distances ($3 < R < 5$), the ratios of the string tension of representations 8, 6, 15-antisymmetric, 10, 27 and 15-symmetric to the one of the fundamental representation are 2.02, 2.21, 3.1, 3.4, 3.8 and 5.6, respectively. This is qualitatively in agreement with the Casimir ratios which are 2.25, 2.5, 4, 4.5, 6 and 7, respectively.

expected behavior of the potential between static quarks, it has some limitations. In particular, it violates the fact that the potential should be always a convex function of distance [10].

Qualitative agreement with Casimir scaling is observed for all the axially symmetric distributions I tried. This is true even if one defines the density to be zero everywhere except on the outer boundary ($r = a$) of the vortex. Figure 2 plots potentials for this distribution with $a = 20$ and $f = 0.1$. Figure 3 shows potentials for the maximally non-axially symmetric core where the flux is zero everywhere except at the two points where the vortex first enters and exits the Wilson loop. A linear regime still exists at intermediate distances but qualitative agreement with Casimir scaling is lost: slope of potentials with larger Casimir operators have smaller string tensions. For example, the string tension for representations 6 and 8 are larger than the ones for representations 15-antisymmetric and 27. In this case the order of potentials at long distances changes as well. For example, the potential for representation 27 is less than the potential for representation 8 in the screened regime. It still

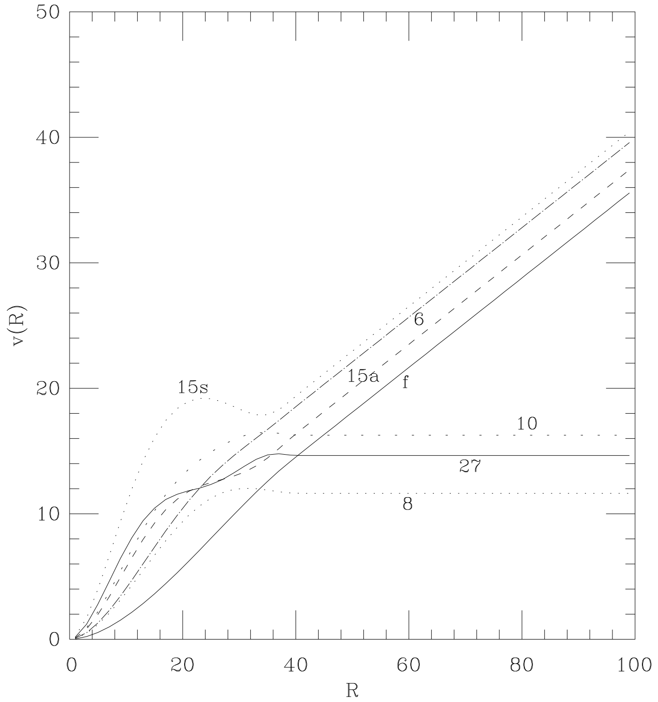

Figure 2: Same as figure 1 but for density distribution equal to zero everywhere except at the vortex circumference($r = a$).

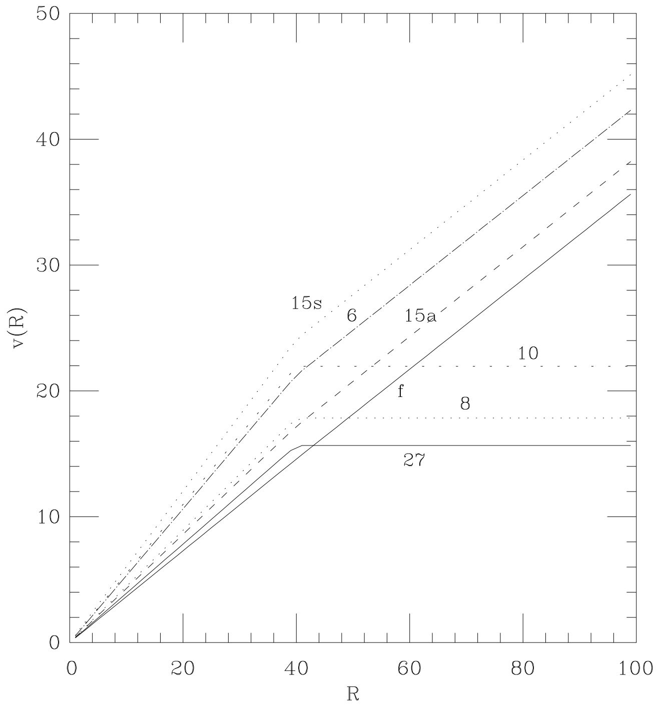

Figure 3: Static sources potential using a density distribution proportional to delta function which is zero every where except at the two points where the vortex first enters and exits the Wilson loop.

remains true, however, that the zero triality representations screen, and non-zero triality ones approach the fundamental string tension at long distances. However the non-convexity of potentials is almost gone. Note that this flux distribution is probably unphysical. We expect the flux in the lowest energy vortex to be axially symmetric. The fact that potentials obtained from physical flux distributions agree qualitatively with Casimir scaling, is the strength of the vortex model. However recent numerical simulation results [8] show that potentials are quantitatively in agreement with Casimir scaling with an accuracy of 5 percent, the feature that is lost in fat-center-vortices results. For example, from figure 2, ratios of potentials for representations 6, 8, 10, 15-symmetric, 15-antisymmetric, and 27 to the fundamental representation are 2, 1.82, 3, 5.4, 2.7, and 3.4, respectively whereas the same ratios for Casimir scaling would be 2.5, 2.25, 4.5, 7, 4, and 6, respectively.

One can get similar results for potentials at intermediate and long distances using one dimensional functions for $\alpha(x)$ similar to those of ref. [3]. I have tried several functions. An example is:

$$\beta(x_v) = \begin{cases} \dfrac{4\pi}{\sqrt{3}} & x_v > a \\ 0 & x_v < -a \\ \dfrac{2\pi}{\sqrt{3}} + \dfrac{2\pi}{\sqrt{3}}\left(\exp\left\{b\left[1 - \dfrac{1}{(x_v/a+1)^2}\right]\right\} - \right. & \\ \qquad\qquad \left. - \exp\left\{b\left[1 - \frac{1}{(x_v/a-1)^2}\right]\right\}\right) & -a < x_v < a. \end{cases} \tag{4.5}$$

$\alpha_R(x_v)$ is then given by eq. (4.3). Note that in this case no density distribution is defined and therefore no integration is needed to find $\beta(x)$. For moderate b this results in a flux profile similar to the one used in ref. [3] and leads to qualitative Casimir scaling. However for b large enough this gives similar plot to figure 3 and violates Casimir scaling. Thus integrating a circularly symmetric distribution inside the Wilson loop is more physical than just picking a function $\beta(x)$ at random. An arbitrary one dimensional distribution is not necessary consistent with axial symmetry, and in the above example, large b certainly does not correspond to any axially symmetric distribution.

5. Conclusion

By applying the fat-center-vortices model to SU(3) and using presumably physical axially symmetric density distributions for the vortex, I showed that for several representations there exists a region at intermediate distances in which the static potential is linear and qualitatively in agreement with Casimir scaling. This is also in agreement with the observation in SU(3) simulations of a linear potential in proportion to Casimir ratio of the representation [4]–[8]. However, the Casimir proportionality is dependent on the flux distribution in the vortex and it is possible to lose this feature by changing the distribution function to a non-axially symmetric distribution. At large distances, zero-triality representations will be screened and the potentials for non-zero triality representations parallel the one for the fundamental representation. Some of the expected features of the screening pattern are also lost for non-axially symmetric distributions. The conclusion is that Casimir scaling and the pattern of screening depend on the detailed vortex structure and are not simple kinematic consequence of the fat center vortex picture. However it is also clear that these properties are rather robust and are likely to survive with physically realistic vortices. On the other hand, potentials are not quantitatively in agreement with Casimir scaling as predicted by recent numerical simulations. This suggests that the fat-center-vortices model needs further refinement if it is to remain viable. In particular, one may need an appropriate physical flux distribution of vortex sizes. Further numerical studies of these issues are in progress.

Acknowledgments

I wish to thank Claude Bernard for his help in this work.

References

[1] G. 't Hooft, *A property of electric and magnetic flux in nonabelian gauge theories*, *Nucl. Phys.* **B 153** (1979) 141.

[2] J.M. Cornwall, *Quark confinement and vortices in massive gauge invariant QCD*, *Nucl. Phys.* **B 157** (1979) 392;
G. Mack and V.B. Petkova, *Comparison of lattice gauge theories with gauge groups* $\mathbb{Z}_2$ *and* SU(2), *Ann. Phys. (NY)* **123** (1979) 442; *Sufficient condition for confinement of static quarks by a vortex condensation mechanism*, *Ann. Phys. (NY)* **125** (1980) 117; $\mathbb{Z}_2$ *monopoles in the standard* SU(2) *lattice gauge theory model*, *Z. Physik* **C 12** (1982) 177;
H.B. Nielsen and P. Olesen, *A quantum liquid model for the QCD vacuum: gauge and rotational invariance of domained and quantized homogeneous color fields*, *Nucl. Phys.* **B 160** (1979) 380;
J. Ambjørn and P. Olesen, *On the formation of a random color magnetic quantum liquid in QCD*, *Nucl. Phys.* **B 170** (1980) 60;
J. Ambjørn, P. Olesen, *A color magnetic vortex condensate in QCD*, *Nucl. Phys.* **B 170** (1980) 265;
L.G. Yaffe, *Confinement in* SU(N) *lattice gauge theories*, *Phys. Rev.* **D 21** (1980) 1574.

[3] M. Faber, J. Greensite and S. Olejník, *Casimir scaling from center vortices: towards an understanding of the adjoint string tension*, *Phys. Rev.* **D 57** (1998) 2603 [hep-lat/9710039].

[4] N.A. Campbell, I.H. Jorysz and C. Michael, *The adjoint source potential in* SU(3) *lattice gauge theory*, *Phys. Lett.* **B 167** (1986) 91.

[5] C. Michael, *Hadronic forces from the lattice*, *Nucl. Phys.* **26** *(Proc. Suppl.)* (1992) 417.

[6] C. Michael, *Adiabatic surfaces from the lattice: excited gluonic potentials*, `hep-ph/9809211`.

[7] S. Deldar, *The string tensions of the* SU(3) *representations*, *Nucl. Phys.* **73** *(Proc. Suppl.)* (1999) 587 [`hep-lat/9809137`]; *Pure* SU(3) *potentials*, *Nucl. Phys.* **83** *(Proc. Suppl.)* (2000) 440 [`hep-lat/9909077`]; *Static* SU(3) *potentials for sources in various representations*, *Phys. Rev.* **D 62** (2000) 34509 [`hep-lat/9911008`].

[8] G.S. Bali, *QCD forces and heavy quark bound states*, `hep-ph/0001312`.

[9] H. Georgi, *Lie algebras in particle physics*, Benjamin/Cummings, Massachusetts, USA 1992.

[10] C. Bachas, *Convexity of the quarkonium potential*, *Phys. Rev.* **D 33** (1986) 2723.